Supplemental Material

Emergence of a spin Hall topological Hall effect in the non-collinear phase of ferrimagnetic insulator terbium-iron garnet

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I. Thickness dependence

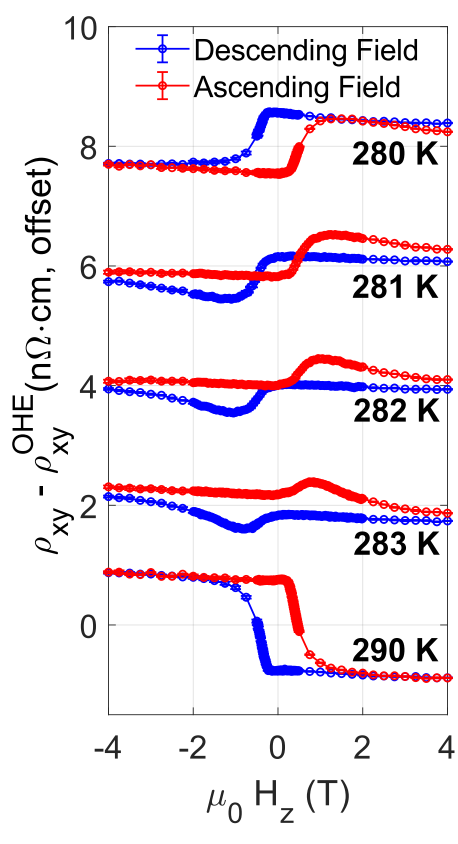

FIG. S1. **Temperature-dependent Hall response of the TbIG(20nm)/Pt heterostructure after subtraction of OHE:** At 280 K and 290 K, the response is dominated by the SH-AHE. Near the magnetic compensation temperature (~282 K), an additional non-monotonic feature associated with SH-THE appears. The SH-AHE reverses sign below and above the compensation point.

In a thicker 20 nm TbIG film, the magnetic compensation temperature shifts to a lower value of approximately 282 K, reflecting the well-known thickness dependence of magnetic compensation in REIG thin films [1]. The temperature window in which the SH-THE signal appears is sharply reduced to only about ±1 K around the compensation point (Fig. S1), and the peak magnitude is smaller than in the 9 nm film. This strong thickness dependence is consistent with the reduced influence of strain in thicker garnet layers, which stabilizes more collinear reversal and limits the formation of the non-trivial magnetic texture associated with the SH-THE response.

II. Geometry dependence

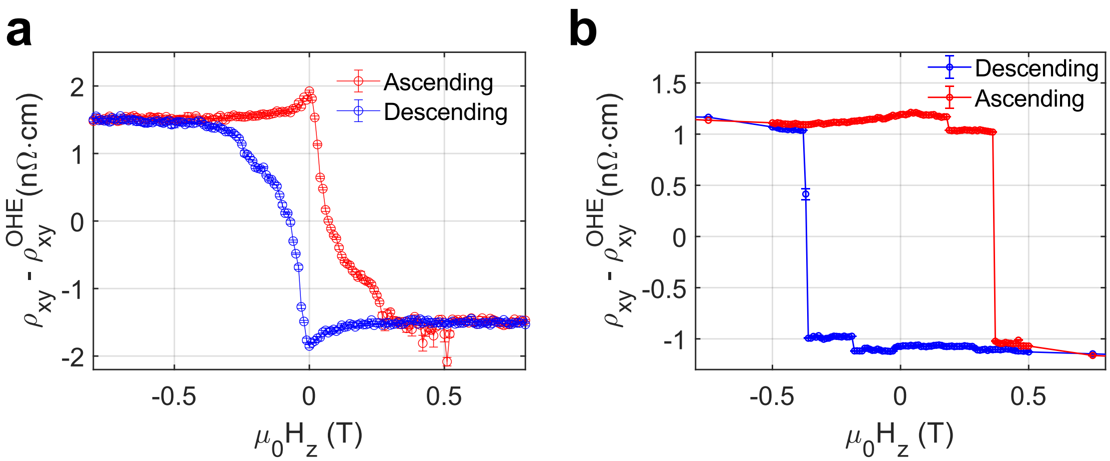

FIG. S2. Comparison of the SH-THE feature in unpatterned and patterned TbIG/Pt films. (a) OHE subtracted Hall signal from the unpatterned film showing a sharp SH-THE peak near zero magnetic field at 320 K. (b) OHE subtracted Hall signal from a lithographically defined 10 μm wide Hall bar measured at the same temperature, for which the SH-THE feature appears broadened into a plateau rather than a narrow peak. Despite the altered shape, both devices exhibit a clear non-monotonic deviation from the OHE + SH-AHE background, indicating that the emergent-field contribution persists after patterning.

Patterning the film into a micron-scale Hall bar introduces additional domain-wall pinning from lithographic edges, process-induced defects, and the confined geometry. This pinning can impede the rapid reversal of canted domains near zero magnetic field, broadening the field range over which the non-trivial spin texture persists and producing the plateau-like SH-THE feature rather than the sharp peak observed in the unpatterned film (Fig. S2). The enhanced pinning also increases the coercive field, consistent with more hindered domain-wall motion during reversal. Importantly, the raw Hall data from the patterned bar still exhibit a clear hump relative to the OHE + SH-AHE background, confirming that the underlying SH-THE response remains robust and that patterning affects only the magnetic field profile, not the presence of the underlying emergent-field-producing texture.

III. Device fabrication and magneto-transport measurements

The as-grown samples were cleaned with acetone and isopropanol prior to patterning. The structures were patterned by photolithography employing a DMO MicroWriter ML3 and a negative photoresist, followed by Ar^+ ion milling for etching. Six terminal Hall bar devices of 10 μm width and 100 μm length were fabricated. For the electrical measurements, the sample was mounted with OOP magnetic field within a variable temperature insert (VTI) installed in a superconducting coil capable of generating magnetic fields up to 12 T. Measurements were conducted in a delta-mode configuration using a Keithley 6221 current source and Keithley 2182A nanovoltmeter for the Hall bar.

IV. Temperature dependence of effective emergent magnetic field

The effective emergent field B_{eff} is estimated using the phenomenological relation $\rho_{xy}^{THE} = R_H \cdot B_{eff}$, for which ρ_{xy}^{THE} is the extracted topological Hall resistivity and R_H is the ordinary Hall coefficient obtained from high-field linear fits. The resulting B_{eff} exhibits a pronounced maximum near the magnetic compensation temperature, consistent with the temperature dependence of the SH-THE signal. The estimate is intended as an order-of-magnitude indication, as the measured Hall response arises from the Pt layer and depends on interfacial spin–charge conversion processes.

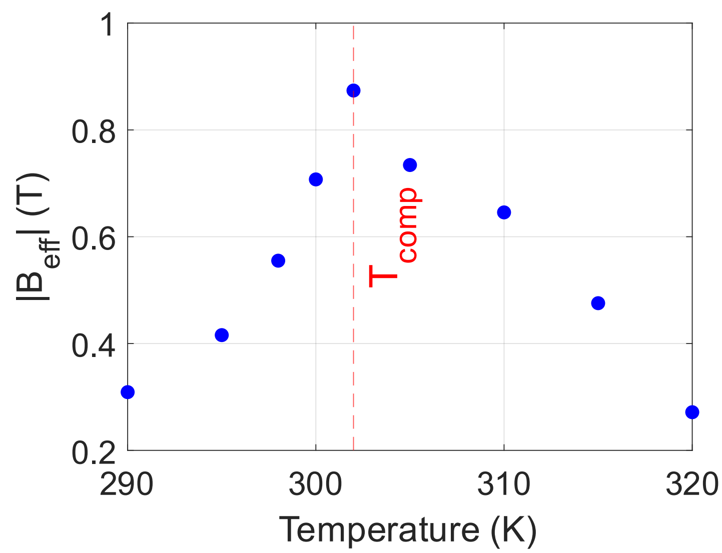

FIG. S3. Temperature dependence of the effective emergent field B_{eff}, estimated from $\rho_{xy}^{THE} = R_H \cdot B_{eff}$ using the extracted topological Hall resistivity and ordinary Hall coefficient.

References

[1] J. M. Liang, X. W. Zhao, Y. K. Liu, P.G. Li, S. M. Ng, H. F. Wong, W. F. Cheng, Y. Zhou, J. Y. Dai, C. L. Mak, and C. W. Leung, The thickness effect on the compensation temperature of rare-earth garnet thin films, Applied Physics Letters **122**, 242401 (2023).