# Generation of super-oscillatory electron beams beyond the diffraction limit

**Authors:** Roei Remez (1), Yuval Tsur (1), Peng-Han Lu (2), Amir H. Tavabi (2), Rafal E. Dunin-Borkowski (2), Ady Arie (1)
1. School of Electrical Engineering, Fleischman Faculty of Engineering, Tel Aviv University, Tel Aviv, ISRAEL
2. Ernst Ruska-Centre for Microscopy and Spectroscopy with Electrons and Peter Grünberg Institute, Forschungszentrum Jülich, Jülich, GERMANY

**DOI: 10.1002/9783527808465.EMC2016.6221**
**Corresponding email:** p.lu@fz-juelich.de
**Keywords:** Super-oscillation, diffraction limit, shaping electron beam

In 1873, Ernst Abbe discovered that the imaging resolution of conventional lenses is fundamentally *limited by diffraction*, which, since then, has been overcome using a variety of different approaches in optical microscopy. In electron microscopy, thanks to remarkable developments in aberration corrected electron optics, the resolution of transmission electron microscopes (TEMs) and scanning TEMs (STEMs) has reached the sub-Ångström regime. However, it is still limited by instrumental stability, residual higher-order aberrations and the *diffraction limit* of the electron-optical system. Recently, a concept termed super-oscillation, which is analogous to the idea of super-directive antennas in the microwave community [1], was proposed [2, 3] and applied in light optics for far field imaging of sub-wavelength, barely-resolved objects beyond the diffraction limit [4]. A super-oscillating function is a band-limited function that is able to oscillate faster locally than its highest Fourier component and thereby produce an arbitrarily small spot in the far field.
Here, we demonstrate experimentally for the first time a super-oscillatory electron beam whose characteristic probe size is much smaller than the Abbe diffraction limit. Figure 1(a) shows scanning electron microscopy (SEM) images of a conventional grating mask (left) and a super-oscillation off-axis hologram (right) that have the same outer diameters (10 μm). The masks were fabricated by focused ion beam milling 200-nm-thick SiN membranes coated with 150 nm Au. The masks were inserted into the C2 aperture plane of a probe-corrected FEI Titan 80-300 (S)TEM. Owing to the probe aberration corrector and relatively small numerical aperture (convergence semi-angle), diffraction-limited spots could be easily obtained from the conventional grating (Fig. 1, left), while a super-oscillatory electron probe, which was generated at the first diffraction order (Fig. 1, right), produced a much smaller hot-spot in the center. The size of the super oscillation hot-spot is approximately one third of that of the diffraction-limited spot. It could theoretically be decreased further, even below the de-Broglie wavelength of the electrons, by varying the ratio between the inner and outer radii.
Further applications of such super-oscillatory electron wave functions, e.g. enhanced STEM imaging, will be presented.

**Acknowledgments:**
This work was supported by the German-Israeli Project Cooperation (DIP) program from the German Research Foundation (DFG) and the Israel Science Foundation, Grant No. 1310/13. RDB thanks the European Research Council for an Advanced Grant.

**References:**
[1] G. Di Francia, Super-gain antennas and optical resolving power, Nuovo Cim. 9, 426 (1952).
[2] Y. Aharonov, J. Anandan, S. Popescu and L. Vaidman, Superpositions of time evolutions of a quantum system and a quantum time-translation machine, Phys. Rev. Lett. 64, 2965-2968 (1990).
[3] M. Berry, Faster than Fourier, in Quantum Coherence and Reality, Celebration of the 60th Birthday of Yakir Aharonov, J. S. Anandan and J. L. Safko, eds. (World Scientific, 1994), pp. 55-65.
[4] E. T. F. Rogers, J. Lindberg, T. Roy, S. Savo, J. E. Chad, M. R. Dennis and N. I. Zheludev, A super-oscillatory lens optical microscope for subwavelength imaging, Nat. Mater. 11, 432-435 (2012).

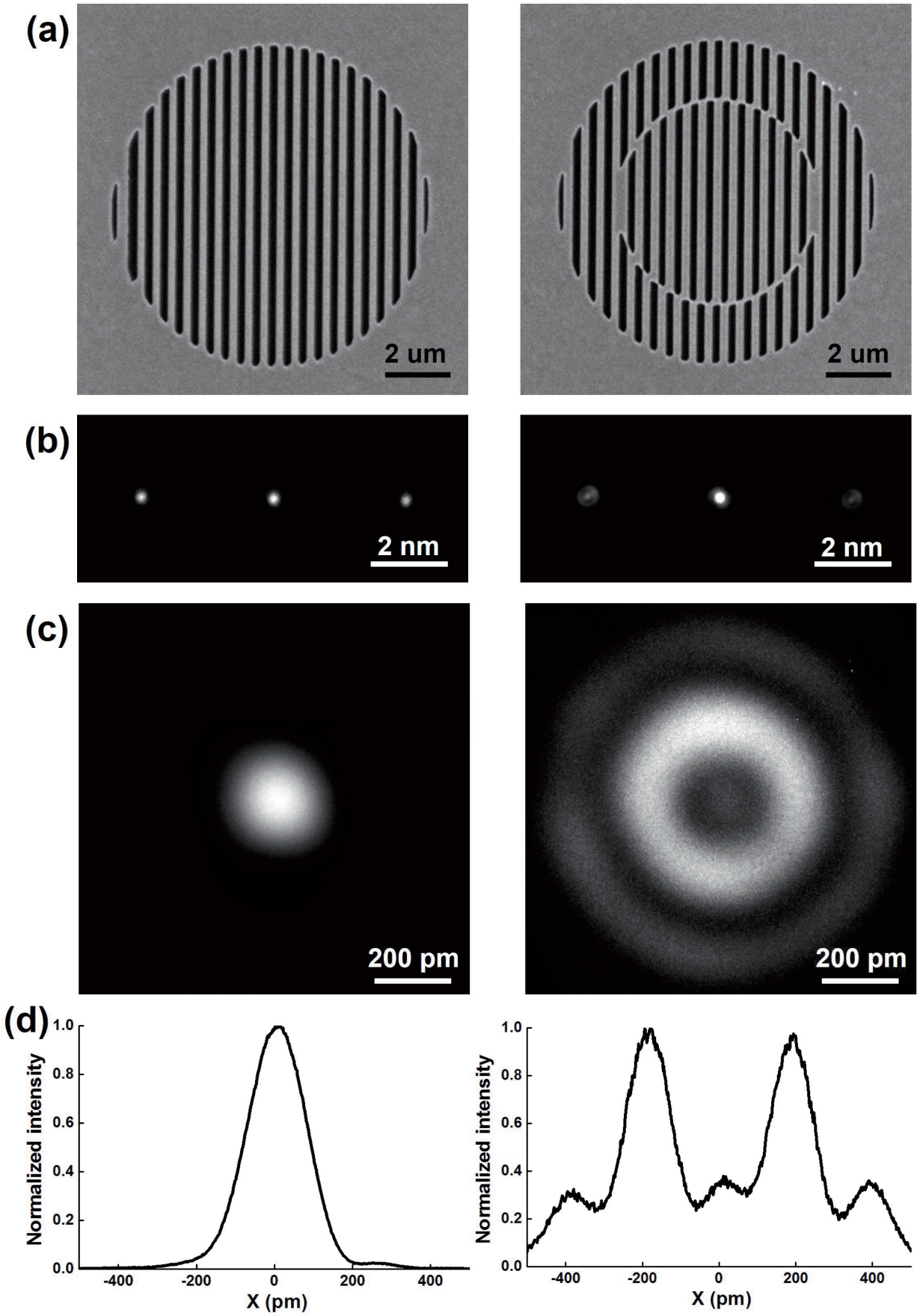


Figure 1. (a) SEM images of a conventional grating mask (left column) and a super-oscillation hologram (right column) with the same outer diameter. (b) Experimentally measured far-field distribution of the diffracted electrons. (c) Experimentally measured diffraction-limited (left) and super oscillatory (right) electron probes created at the first diffraction order. (d) Normalized intensity of the transverse distribution of (c), showing that the size of the super-oscillation central hot-spot is approximately one third of the size of the corresponding diffraction-limited spot.