REAL SPACE SOLUTION OF THE INTENSITY TRANSPORT EQUATION FOR STRONG PHASE OBJECTS

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While noise levels have frustrated its exploitation at high resolution, the 'intensity transport' approach[1,2] appears promising at lower resolution for the determination of large phase shifts such as arise for specimens with varying electric potential or magnetisation. Transverse (x, y) derivatives of the phase ϕ of a wave are related to the forward (z) intensity gradient (easily observed via two images with a small defocus difference) by the equation

$$\nabla \cdot (I \nabla \phi) = \frac{2\pi}{\lambda} \frac{\partial I}{\partial z},$$

which linearises a notoriously non-linear inverse problem, and has a unique solution for ϕ for a given (or assumed) 'boundary' phase at the periphery of the field of view. A fast and simple Fourier space approach provides a *periodic* solution when there is no transverse intensity variation (i.e. I is constant)[2,3]; as this seems very restrictive, we have explored a real-space approach that accommodates intensity variations and allows the boundary phase to be specified arbitrarily. Approximating the derivatives by finite differences, and adopting the 'relaxation' approach to the differential equation, we refine a scaled phase estimate iteratively according to

$$\phi_{ij}^{n+1} = J_{ij} + \frac{(I_{i+1j} + I_{ij})\phi_{i+1j}^{n} + (I_{i-1j} + I_{ij})\phi_{i-1j}^{n} + (I_{ij+1} + I_{ij})\phi_{ij+1}^{n} + (I_{ij-1} + I_{ij})\phi_{ij-1}^{n}}{4[I_{ij} + I_{i+1j} + I_{i-1j} + I_{ij+1} + I_{ij-1}]}$$

where J denotes the intensity *difference* $2\pi[I(z+\Delta z) - I(z-\Delta z)]$ and the intensities are normalised to have unit mean. This is applied to all pixels in raster-scan order, and the process repeated a few thousand times – taking something like twenty minutes for a 512^2 field ; the result should be multiplied by $\Delta x^2 / \lambda \Delta z$ to yield the true (unscaled) phase, so these intervals must be determined somehow.

We have applied this approach to each member of a focal series of images of a ferromagnetic Co film about 15nm thick (fig.1), recorded with a CCD camera in a 200kV FEGTEM with a Lorentz lens. Contrast is high (maximum ~10 times the mean); focus (*un*calibrated) extends a millimetre or so either side of gaussian focus, in uneven steps; the magnification [roughly 40K×, and accurately calibrated] varies by a factor of two through the series, and images were mutually registered before processing (in magnification as well as orientation and position) using an old approach due to Saxton[4]. The phase deduced from the two images either side of gaussian focus, with a boundary value of zero, is shown in fig.2, and fig.3 shows one component of the magnetic flux density inferred, $B_x = (\hbar / et)\partial\phi / \partial y$ for thickness t). Fig.4 shows the flux density calculated via the Fourier approach – remarkably similar. The flux density proved close to uniform within domains, as expected.

We have sought to test the approach critically by comparing the waves $\psi = \sqrt{I}\exp(i\phi)$ recovered at each focus level, which should be inter-convertible by simple forward or

backward propagation. We had also hoped to establish the scaling factor $\Delta x^2 / \lambda \Delta z$ via this comparison, as the intervals appear in the same combination in phase scaling factors and in refocusing phase shifts. However at this stage, we have *not* found the waves convincingly consistent: those from planes near gaussian focus appear consistent on rescaling so as to have a range of about 35rad, but those from farther planes do not.

We hope to explore further the influence of the unknown boundary phase (which simulations suggest is not too serious); possible acceleration of the solution – by over-relaxation or by successively decreasing the sampling interval following an initial low resolution solution; and the use of additional constraints such as constancy of the flux density across the field of view.

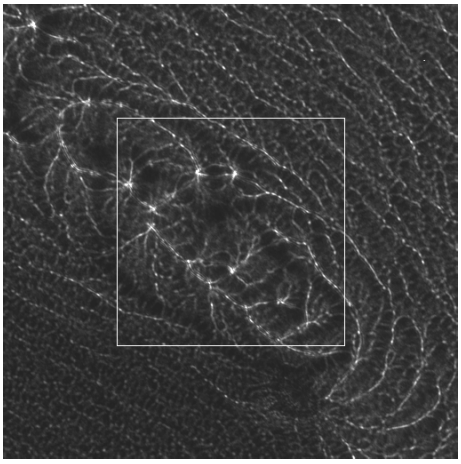

Fig.1 Image of Co film, one step under focus; the 2.6μm square field marked is that used in later figures.

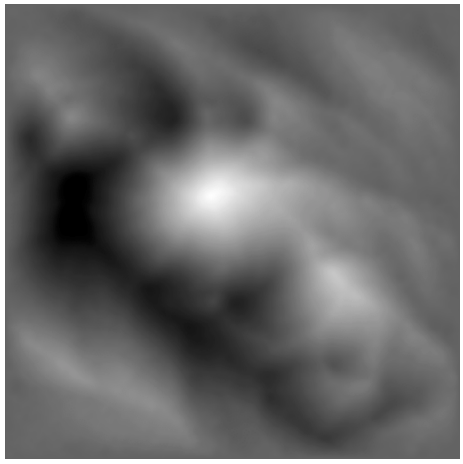

Fig.2 The phase recovered at gaussian focus for the image in fig.1; range approximately 35rad.

Fig.3 *x*-component of projected flux inferred from the phase in fig.2; approximate max. density 2T.

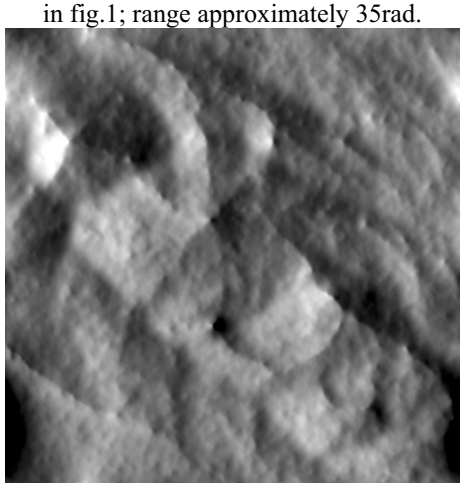

Fig.4 *x*-component of projected flux density inferred from phase recovered by Fourier approach.

References

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